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The fluid mechanics of the ureter with an inserted catheter

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The effect of the presence of a catheter upon the pressure distribution inside the ureter is considered. Under the assumption of Stokes flow and long wavelengths it is shown that during the contraction a thin lubrication-type layer is formed between the catheter and the ureteral wall, capable of sustaining high pressures. Furthermore, it is found that the insertion of a catheter does not change the pressure distribution inside the ureter appreciably, leading to the conclusion that a urometrogram obtained with a catheter gives a good representation of the pressure inside an undisturbed ureter.

1. Introduction

The pressure distribution inside the ureter, measured as a function of time (the urometrogram), is one of the important diagnostic tools in urology. Such pressure measurements are obtained through insertion of à catheter inside the ureter. Many physiologists have wondered about the effect of this catheter on the pressure distribution and have asked themselves whether the observed urometrogram is a fair representation of the pressure distribution in an undisturbed ureter. Some have gone as far as to question the validity of their measurements.‡

Recently, Lykoudis & Roos (1970) theoretically analyzed the fluid mechanics of the ureter, putting special emphasis on the urometrogram.§ They showed that the ureter has to collapse to very small diameters, of the order of 0·05 mm, in order to produce pressures of 25 mmHg, which is the level normally observed. On the other hand, the catheters used to measure these pressures have diameters of the order of 1 mm. They interpreted this apparent inconsistency by postulating the existence of a thin layer of fluid between the wall of the ureter and the catheter capable of sustaining the observed high pressures. This lubrication film would behave similarly to the ones in load-carrying bearings.

In order to provide a better understanding of this problem, we will investigate the case of a peristaltic wave moving over a stationary cylinder. The cylinder is assumed to be infinitely long, excluding the influence of the tip of the catheter. This is not regarded as a major limitition since ureteral catheters have side

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‡ See, for example, the discussion in Boyarsky (1970).
§ See also Roos (1970).

openings, in contrast to heart catheters which have an open end. The shape of the wave is taken to be similar to the one used by Lykoudis & Roos (1970) in their analysis of the ureter.

2. Mathematical formulation

Let us assume an infinitely long cylinder of radius δ inside a peristalting tube of which, in a co-ordinate system moving with the wave at speed c, the collapsing part is given by (see figure 1)

$$h = b + a(x/\lambda_1)^n, \tag{1}$$

where $b > \delta$ and $\lambda_1 \gg a+b$. The relaxation part is approximated by a step function, although of course in reality a smooth relaxation is required. The relaxed section is taken to be a straight tube.

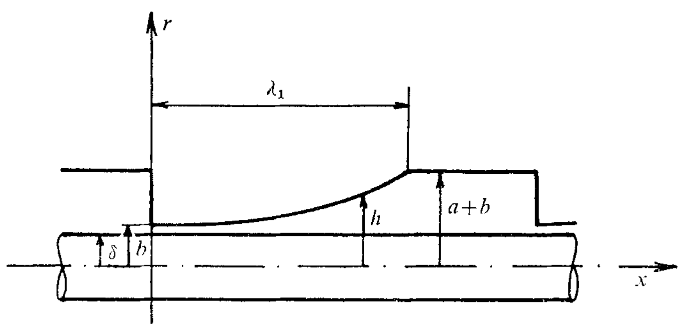

FIGURE 1. Co-ordinate system and geometry.

Since the Reynolds number for the type of flow in the ureter is of order 1 or smaller, one can neglect the inertia terms in the equations of conservation of momentum. Realizing further that the wavelength is much larger than the average diameter, the equations governing this problem in the moving co-ordinate system are the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \tag{2}$$

and the equations of conservation of momentum

$$\frac{\partial p}{\partial x} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{3}$$

$$\partial p/\partial r \simeq 0 \quad \text{or} \quad p \simeq p(x). \tag{4}$$

In this co-ordinate system the boundary conditions are

$$u = -c \quad \text{at} \quad r = \delta, \tag{5}$$

$$u = -c \quad \text{at} \quad r = h(x), \tag{6}$$

$$v = 0 \quad \text{at} \quad r = \delta. \tag{7}$$

Since the pressure is a function of x only, (3) can be integrated immediately using the boundary conditions (5) and (6). This leads to an axial velocity distribution of the following form:

$$u = -c + \frac{1}{4\mu}\frac{dp}{dx}\left((r^2-\delta^2)-(h^2-\delta^2)\frac{\ln(r/\delta)}{\ln(h/\delta)}\right). \tag{8}$$

In the above equation, which is similar to the one for the flow through an annulus, the local pressure gradient is still an unknown quantity. Following the method used in lubrication theories (Schlichting 1960), this pressure gradient can be expressed in terms of the flux q, which is given by

$$q = 2\pi\int_\delta^h ur\,dr. \tag{9}$$

Substitution of (8) and integration results in a pressure gradient

$$\frac{dp}{dx} = -\frac{8\mu}{\pi}\left(\frac{q}{h^2-\delta^2}+\pi c\right)\frac{\ln(h/\delta)}{\ln(h/\delta)(h^2+\delta^2)-(h^2/\delta^2)}. \tag{10}$$

By integrating (10), one obtains the pressure distribution along the x axis

$$p_x = p_0 - 8\mu c\int_0^x \frac{dx}{(h^2+\delta^2)-(h^2-\delta^2)/\ln(h/\delta)} - \frac{8\mu q}{\pi}\int_0^x\frac{dx}{(h^4-\delta^4)-(h^2-\delta^2)^2/\ln(h/\delta)}. \tag{11}$$

The unknown flux q is found by setting a condition on this pressure distribution. In accordance with urological observations, it is assumed that there does not exist an overall pressure gradient over one wavelength for the normal ureter or

$$\Delta p_\lambda = p_\lambda - p_0 = \int_0^\lambda \frac{dp}{dx}\,dx = 0. \tag{12}$$

However, the diameter of the relaxed part of the wave is large compared to that of the collapsed section. Therefore the pressure-drop along this relaxed part is negligibly small and condition (12) can be substituted by

$$\Delta p_{\lambda_1} = p_{\lambda_1} - p_0 = 0. \tag{13}$$

With this condition, the following expression for q is obtained:

$$q = -\pi c\left(\int_0^{\lambda_1}\frac{dx}{(h^2+\delta^2)-(h^2-\delta^2)/\ln(h/\delta)}\right)\Bigg/\left(\int_0^{\lambda_1}\frac{dx}{(h^4-\delta^4)-(h^2-\delta^2)^2/\ln(h/\delta)}\right). \tag{14}$$

In contrast to the case without the inserted cylinder, algebraic solutions of the above integrals and thus of the pressure distribution are not readily obtainable. For this reason, they need to be evaluated numerically for each particular case.

3. Numerical pressure and velocity distributions

Ureteral pressure studies are normally made with catheters ranging in diameter from 1–2 mm. Our numerical study was performed for the extreme case of 2 mm ($\delta = 1$ mm) such that any effect, if existing and attributed to the inserted cylinder, would become clearly visible. For purposes of comparison, the shape of the wave was kept similar to the one used by Lykoudis & Roos (1970). Figure 2 shows the obtained numerical pressure distribution, together with an

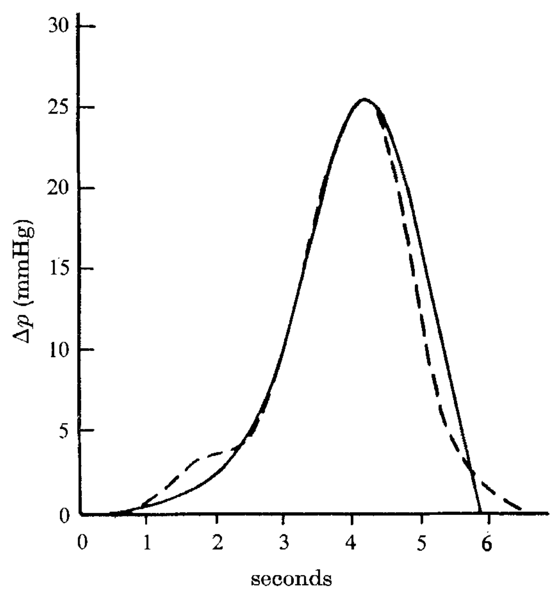

FIGURE 2. Experimental and theoretical urometrograms as obtained from the model with inserted catheter. a = 1·35 mm, δ = 1 mm, $b-\delta$ = 0·0195 mm, c = 30 mm/sec, λ_1 = 225 mm, n = 4: ——, present theory; – – –, experiment (Kiil 1957, p. 59).

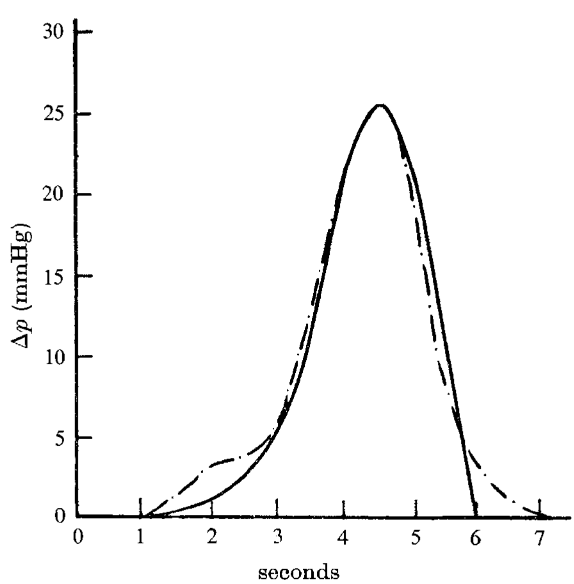

FIGURE 3. Experimental and theoretical urometrograms as obtained from the model by Lykoudis & Roos (1970). a = 2·35 mm, b = 0·02 mm, c = 30 mm/sec, λ_1 = 250 mm, n = 4: ——, theory; —·—, experiment (Kiil 1957, p. 59).

experimental urometrogram taken from Kiil (1957). It is seen that they coincide very well except for the tail, which section is not very well covered by the theory.

Figure 3, which is taken from Lykoudis & Roos (1970), shows the pressure distribution obtained from the theory without the inserted cylinder and the same experimental urometrogram. Comparison of these figures shows that both theoretical pressure distributions are very similar. The wave speed, the maximal diameter, and the shape parameter n are identical in both cases, while the contraction length λ_1 differs only by a small amount. The minimal clearance

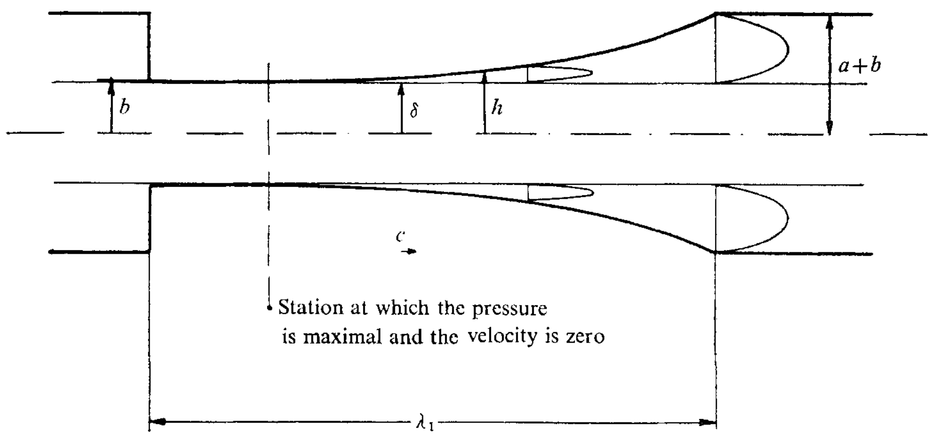

FIGURE 4. The theoretical axial velocity distribution as obtained from the model with inserted catheter. $a = 1{\cdot}35$ mm, $\delta = 1$ mm, $b-\delta = 0{\cdot}0195$ mm, $c = 30$ mm/sec, $\lambda_1 = 225$ mm, $n = 4$.

between the wall and the cylinder needed to obtain the required maximum pressure is found to be of the same order of magnitude as the minimal radius needed in the theory without the inserted cylinder. Both are about 0·02 mm. This result shows that the interpretation made by Lykoudis & Roos (1970), leading to the idea of the thin lubrication-type layer between the ureter wall and the catheter during the collapsing phase, is correct.

The axial velocity distribution is given in figure 4. Just as in the theory for the ureter without the inserted catheter, we find that the point of zero axial velocity or maximum pressure lies ahead of the point of maximal constriction. Comparison with experimental data is impossible, since such measurements cannot be made.

4. Conclusion

It has been shown that the contraction of a peristaltic wave around a catheter results in the forming of a thin lubrication-type layer which can sustain high pressures. In the absence of the catheter, the walls of the peristaltic tube collapse upon themselves, leaving a very small cylinder of fluid sustaining these high pressures. Also, the insertion of a catheter in a peristaltic tube does not produce substantial changes in the pressure distribution, although the velocity field will change completely. During a contraction the muscles in the ureteral wall are performing work against the rising pressure. This work is limited by the amount of stored energy before the contraction. Therefore, in both cases the maximum pressure will be about the same, given a similar time rise in pressure.

From this it can be concluded that the urological pressure measurements made with a catheter give a good representation of the pressure distribution in the undisturbed ureter as long as the catheter does not block the ureter during the relaxation phase.

REFERENCES

BOYARSKY, S. (Ed.) 1970 *Hydrodynamics of the Ureter*. Proc. of the Workshop on the Hydrodynamics of the Upper Urinary Tract. New York: Academic.

KIIL, F. 1957 *The Function of the Ureter and Renal Pelvis*. Philadelphia: Saunders.

LYKOUDIS, P. S. & ROOS, R. 1970 The fluid mechanics of the ureter from a lubrication theory point of view. *J. Fluid Mech.* **43**, 661.

ROOS, R. 1970 Peristaltic transport in Physiological systems and the ureter in particular. Ph.D. dissertation, Purdue University.

SCHLICHTING, H. 1960 *Boundary Layer Theory*, 4th ed. New York: McGraw-Hill.